1999 年教學設計獎勵計劃

圓的基本性質

學科名稱：數學

適合程度：高一

一九九九年教學設計

題目　　　　：圓的基本性質。

適合程度／班別：高中一／英中四年級。

設計科目　　：數學〔平面幾何〕。

設計目標　　：1. 學習圓內角的性質和其有關的定理。
2. 研究圓內接四邊形的性質及有關的定理。
3. 學習圓切線的性質及有關的定理。
4. 學習外接圓等。

設計內容　　：透過在課堂活動中〔本澳韋輝樑先生所提的數學實驗教學法〕，使用「尋找模式」的思考方法，學生應發覺：

1. 圓內角的性質。
2. 圓內接四邊形的性質。
3. 觀察弦變為切線的過程，藉此讓學生明白切線的性質。
4. 強調圓心位於弦的垂直平分線上。

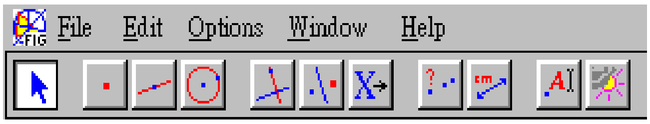
計劃特色　　：

1. 嘗試用數學實驗法來向學生引入圓的基本性質。
2. 藉此課題對澳門數學實驗教學的過程及教學法研究作初探。
3. 教師的實驗過程指引及學生實驗報告紙有中英文版本，適合本澳中英文學教師採用。

我們將使用數學軟件 **Cabri Geometry** 來進行數學實驗活動，現時所提供的軟件是試用版，每隔 15 分鐘便會自動停止運作，但已提供足夠時間為教師作課堂展示或學生進行一系列作圖部驟去發現幾何定理。
原廠 **Cabri** 套裝可在香港購買，時價為港幣 1480 元。

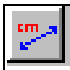
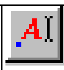
供應商地址：
Hong Kong Electronic (OA) Centre
香港九龍旺角登打士街 43A 地下
電話：23856603

以下部份是協助教師和學生去熟習 **CABRI** 軟件的作圖操作情況，如果教師安排學生每人一機在電腦上作有關數學實驗時，應先讓學生進行以下練習，本練習可令學生學習 **CABRI** 的各項作圖功能，同時還可透過作圖操作加強對一些基本數學名稱的理解，學生透過各項練習步驟對數學幾何定理、公理及概念上更加理解，明白其中的意義。教師可向學生派發以下的練習紙，當學生在作圖上遇到困難時，便可作為參考資料。



Pointer **Rotate** **Dilate** **Rotate and Dilate**	**Point** **Point On Object** **Intersection Points**	**Line** **Segment** **Ray** **Vector** **Triangle** **Polygon** **Regular Polygon**	**Circle** **Arc** **Conic**

Perpendicular Line **Parallel Line** **Midpoint** **Perpendicular Bisector** **Angle Bisector** **Vector Sum** **Compass** **Measurement Transfer** **Locus** **Redefine Object**	**Reflection** **Symmetry** **Translation** **Rotation** **Dilation** **Inverse**	**Initial Objects** **Final Objects** **Define Macro**	**Collinear** **Parallel** **Perpendicular** **Equidistant** **Member**

		
Distance and Length	**Label**	**Hide / Show**
Area	**Comments**	**Colour**
Slope	**Numerical Edit**	**Fill**
Angle	**Mark Angle**	**Thick**
Equation and Coordinates	**Fix / Free**	**Dotted**
Calculate	**Trace On / Off**	**Modify Appearance**
Tabulate	**Animation**	**Show Axes**
	Multiple Animation	**New Axes**
		Define Grid

1. 作點及標註點：

1. 在功具箱選取 **Point**。
2. 隨意將光標移到任何位置，然後按滑鼠一下，同時輸入該點名稱(例如A, B, C)，重覆此步驟作另外五個點。
3. 如果你忘記在點上作標註，可在功具列選取 **Label**，使光標移到該點上，當信息 ***This point*** 出現時，將滑鼠按一下，文字區便會出現，輸入該點名稱(例如：A, B, C)，完成後將滑鼠在文字區外按一下。將其他未標註的點重覆作以上步驟。

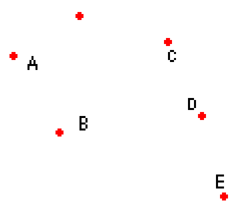


Figure 1.1

2. 改正錯誤：

1. 在功具箱選取 **Pointer**。
2. 將光標移到要刪除的點或線上，當相應的信息出現時(例如：***This point***)，按一下滑鼠來選取圖像，(被選取的圖像便會開始閃動)，按下 DELETE 鍵便可刪除所選取的圖像。

3. 清除畫面：

1. 在功能列的 **Edit** 下選取 **Select All**，然後按 DELETE 鍵。

或 2. 同時按下 CRTL 鍵和 A 鍵，然後按 DELETE 鍵。

4. 作兩條相交線：

1. 在功具箱選取 **Line**。
2. 任意將光標移到其他位置按滑鼠一下，輸入該點名稱(例如 A, B, C)。
3. 將光標移到另一位置，然後按滑鼠一下，就可以拉出一條射線。
4. 重覆以上三個步驟作另一條射線，並與第一條射線相交。
5. 在功具箱選取 **Intersection Point**。
6. 將光標移到兩條射線的交點上，當信息 ***Point at this intersection*** 出現時便按滑鼠一下，輸入該交點名稱。
7. 在功具箱選取 **Point On Object.**。
8. 將光標移動直至信息 ***On this line*** 出現，按滑鼠一下，輸入該點名稱。.

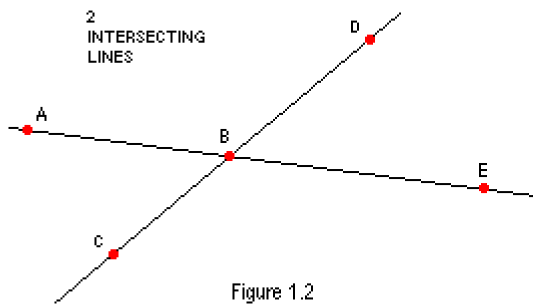


Figure 1.2

5. 清除畫面。

6. 作角：

1. 在功具箱選取 **Ray**。
2. 在視窗中定一角的頂點位置，輸入 B。
3. 將光標移到另一位置，按滑鼠一下作出一條角邊。
4. 將光標移到角的頂點上，當信息 ***This point*** 出現時按滑鼠一下。

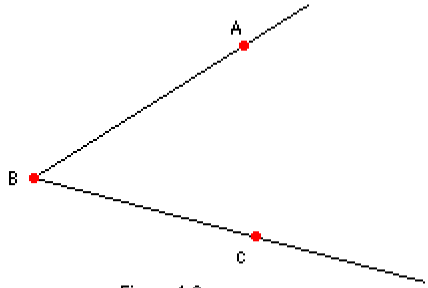


Figure 1.3

5. 將光標移到另一位置按滑鼠一下，便可作出一個鈍角的第二條邊。
6. 在功具箱選取 **Point On Object**。
7. 將光標移到一條射線上，當信息 ***On this ray*** 出現現時便按滑鼠一下，輸入 A。
8. 將光標移到第二條射線上，當信息 ***On this ray*** 出現時便按滑鼠一下，輸入 C。

7. 量度角度：

1. 在功具箱選取 **Angle**。
2. 將光標移到 A 點上，當信息 ***This point*** 出現時按滑鼠一下。
3. 將光標移到 B 點上，當信息 ***This point*** 出現時按滑鼠一下。
4. 將光標移到 C 點上，當信息 ***This point*** 出現時按滑鼠一下。
5. 角度的顯示區將會在視窗中出現，可輸入角的名稱(例如 ABC=45.5)。

8. 改變角的大小：

1. 在功具箱選取 **Pointer**。
2. 將光標移近一條射線上直至信息 **This ray** 出現。
3. 在角的這條邊上按著滑鼠。
4. 拖曳滑鼠便何看到所顯示的角度在改變。

9. 平分一角：

1. 由功具箱選取 **Angle Bisector**.。
2. 將光標移到 A 點上，當信息 ***This point*** 出現時按滑鼠一下。
3. 將光標移到 B 點上，當信息 ***This point*** 出現時按滑鼠一下。
4. 將光標移到 C 點上，當信息 ***This point*** 出現時按滑鼠一下。

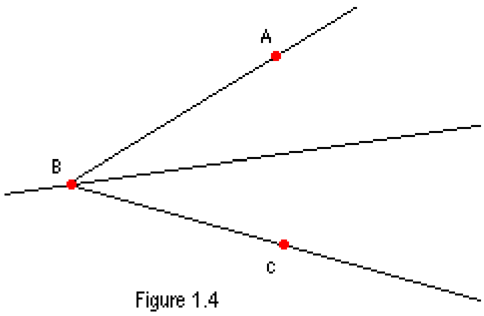


Figure 1.4

5. 在此角的內平分線上作一點。
6. 從此點量度角平分線與角其中一邊所形成的角。
7. 量度角讀數是原來角的一半。

10. 清除畫面。

11. 作平行線：

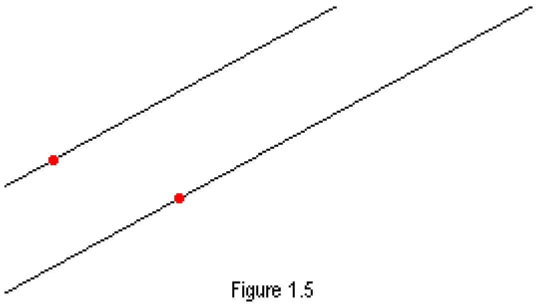


Figure 1.5

1. 在功具箱選取 **Line**。
2. 隨意將光標移到任何位置按滑鼠一下。
3. 將光標移到另一位置，然後按滑鼠一下便可作出一條射線。
4. 在功具箱選取 **Parallel Lines**。
5. 將光標移到該射線上直至信息 ***Parallel to this line*** 出現，然後按滑鼠一下。
6. 將光標移到線外的區域按滑鼠一下便作了一條平行線。

12. 清除畫面。

13. 作垂線：

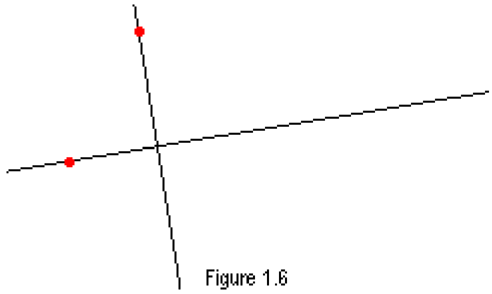


Figure 1.6

1. 在功具箱選取 **Line**。
2. 隨意將光標移到任何位置按滑鼠一下。
3. 將光標移到另一位置，然後按滑鼠一下便可作出一條射線。
4. 在功具箱選取 **Perpendicular Lines**。
5. 將光標移到該射線上直至信息 ***Perpendicular to this line*** 出現，然後按滑鼠一下。
6. 將光標移到線外的區域按滑鼠一下便作了一條垂線。

14. 清除畫面。

15. 作一三角形並標註其頂點：

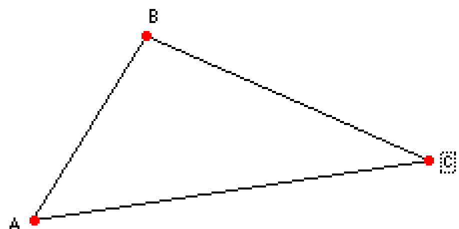
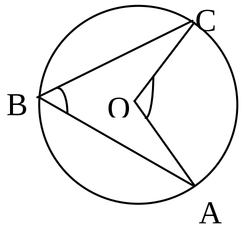


Figure 1.7

1. 在功具箱選取 **Triangle**。
2. 按滑鼠按一下，然後輸入 A，將滑鼠拖曳。
3. 在線段尾按滑鼠一下便可作出三角形的一條邊，輸入 B，再拖曳滑鼠便完成三角形的作圖，最後輸入 C。

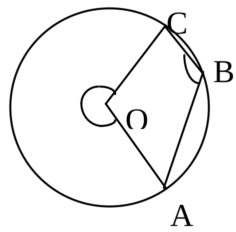
實驗工作紙（1）

圓心角和圓周角



目的：發現圓心角和圓周角之間的關係。

作圖：
1. 作一圓形；
2. 在圓周上任意作 A, B, C 三點；
3. 定 O 為圓心；
4. 連接線段 OA,OC, AB 和 BC；
5. 標註和量度∠AOC 和∠ABC。



∠AOC 稱為**圓心角**，而∠ABC 則是其所對應的**圓周角**。

觀察／實驗：
1. 在圓周上拖曳 A, B 和 C 各點，觀察∠AOC 和∠ABC 的改變，並將結果填寫下表：

∠*AOC*	20°										
∠*ABC*	10°										

2. 你發現∠AOC 和∠ABC 有甚麼關係？當 AOC 成一直線時，你有什麼發現呢？

猜想：
寫出你所作的假設：

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討論：
與同學討論所得結果。

證明（邏輯證明）：
己知：∠AOC 是圓心角，
　　　∠ABC 是其所對應的圓周角。
求證：∠AOC=2∠ABC。

教學指引(1)

<u>題目</u>：發現圓心角和圓周角之間的關係。

<u>基本知識</u>：學生己認識*圓心角*和*圓周角*的定義。

<u>教學時間</u>：約 15 分鐘。

<u>作圖應注意</u>：

Cabri 沒有量度反角∠AOC 的功能，但可運用 360° – ∠AOC 求得。

<u>探討/假設/討論應注意</u>：

a) 因 Cabri 在量度角度是取接近值的關係，所以學生所作的猜想可能是圓心角大概是其圓周角的兩倍。在此情況下，教師可向學生解釋在日常生活中有關近似值的問題。就如∠AOC 是 21.2º，而∠ABC 卻是 10.6º。

b) 指導學生檢查當∠AOC 為反角時的情況。

c) 猜想中可能包括：

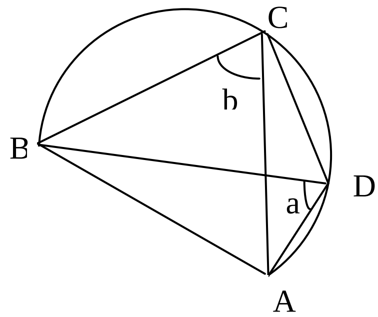
(1) 圓心角是圓周角的兩倍。

(2) 若圓心角是 180º，則圓周角是一個直角。

(3) 半圓上的圓周角是一個直角。

實驗工作紙（2）

同弓形內的圓周角



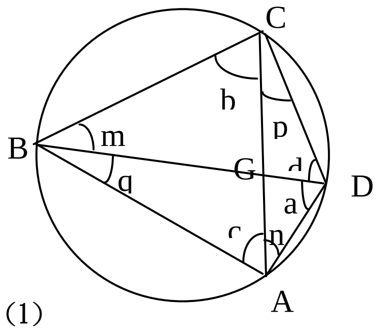
目的：

發現同弓形內的圓周角之間的關係。

∠ADB 和∠ACB 是同**弓形內的圓周角**。

作圖：

1. 作一圓形；
2. 在圓周上任意作 A, B, C 和 D 四點；
3. 連接線段 AB, BC, CD, DA, BD 和 AC；
4. 標註及量度圖中 8 個角的大小。



圖(1)

觀察／實驗：

1. 在圓周上拖曳 A, B, C 和 D 各點，觀察 8 個角的大小改變，並將結果填寫下表。

a	60°										
b	60°										
c	55°										
d	55°										
m	35°										
n	35°										
p	30°										
q	30°										

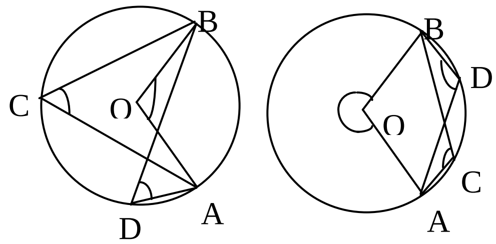
2. 你發現這 8 個角之間有甚麼關係？你可作出什麼結論呢？

猜想：

寫出你的假設。

討論後和再觀察

1. 與同學討論所得結果，並且運用圓心角是圓周角的兩倍這種性質去證明你的假設，將邏輯證明寫在下面。



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2. 在圖(1)，可否發現任何相似三角形呢？

教學指引（2）

<u>**題目**</u>：發現同弓形內的圓周角之間的關係。

<u>**基本知識**</u>：學生已認識*同弓形內的圓周角*定義。

<u>**教學時間**</u>：約 20 分鐘。

<u>**探討／假設應注意**</u>：

(a) 猜想中可包括：

(1) 同弓形內的圓周角皆相等。

(2) $a = b$ ，$c = d$ ，$p = q$ ，$m = n$ 。

(b) 部份學生可能作出以下的猜想：

(1) $a + b + c + d + p + q + m + n = 360^\circ$。

(2) $a + d + m + q = 180^\circ$, $b + c + n + p = 180^\circ$。

教師可指導學生對以上的性質*(1)*和*(2)*在任意四邊形上作檢驗，他們可發現性質*(2)*在任意四邊形上是不成立的。

<u>**討論後和再觀察應注意**</u>：

(a) 證明如下：

$\angle AOB = 2\angle ACB$ (圓心角是圓周角兩倍)

$\angle AOB = 2\angle ADB$ (圓心角是圓周角兩倍)

因此， $\angle ACB = \angle ADB$ 。

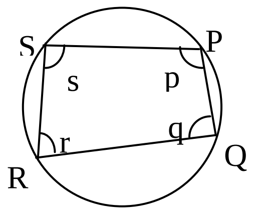
(b) 教師可指導學生在圖(1)中發現任何相似三角形，有兩對相似三角形 i.e. $\Delta BGC \approx \Delta AGD$ 和 $\Delta BGA \approx \Delta CGD$ 。

實驗工作紙（3）

圓內接四邊形

目的：

發現圓內接四邊形的性質。



作圖：

1. 作一圓形；
2. 在圓周上任意作 P, Q, R 和 S 四點；
3. 連接線段 PQ , QR , RS and SP；
4. 標註和量度 p, q, r 和 s 角。

圓內接四邊形是指一四邊形的頂點分別位於（內接於）一圓的圓周上。

觀察／實驗：

1. 在圓周上拖曳 P, Q, R 和 S 各點，觀察 p, q, r 和 s 各點的大小改變，並將結果填寫下表。

p	107°							
q	102°							
r	73°							
s	78°							

2. 你發現 p, q, r 和 s 角之間有甚麼關係？這些關係在任意四邊形上是否成立？圓內接四邊形內角有甚麼性質？

猜想：

寫出你的假設。

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討論後和再觀察：

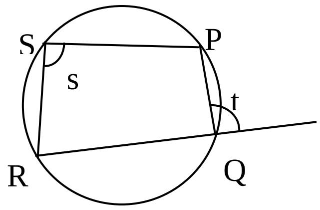
1. 與同學討論所得結果。
2. 圖中，s 和 t 兩角之間有甚麼關係？試運用上面發現的結果去解釋你的假設，並使用 Cabri 去驗證。

3. 證明（邏輯證明）：

己知：PQRS 為一圓內接四邊形。

求證：s = t。

證明：



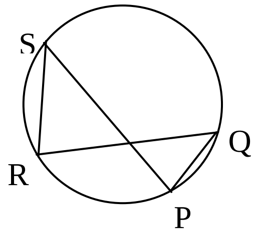
教學指引 (3)

<u>題目</u>：發現圓內接四邊形的性質。

<u>基本知識</u>：學生己認識*圓內接四邊形*的定義。

<u>教學時間</u>：約 20 分鐘。

<u>探討／假設應注意</u>：



(a) 若 P, Q, R, S 各點被拖曳至一個不是圓內接四邊形（見右圖），則會成為同弓形內的圓周角相等情況。

(b) 猜想中可包括：

(1) $p + q + r + s = 360°$。

(2) $p + r = 180°$ ， $q + s = 180°$。

(3) 圓內接四邊形兩對角互補。

教師可指導學生對以上*(1)*， *(2)*和*(3)*的性質在任意四邊形上作檢驗，他們可發現性質*(2)*和*(3)*在任意四邊形上是不成立的，故此教師可提出圓內接四邊形是屬有以上性質的一種特別四邊形。

<u>討論後再觀察應注意</u>：

學生應能運用以下的性質去解釋圓內接四邊形外角（t）與其內對角相等：

(i) 圓內接四邊形的兩對角互補；

(ii) 直線上的鄰角互補。

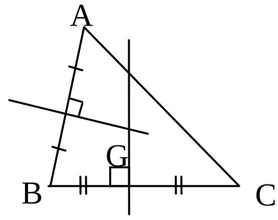
實驗工作紙（4）

三角形外接圓

目的：
發現三角形三邊垂直平分線的性質。

利用 Cabri 的簡單指令，作一個通過三角形三個頂點的圓形。

作圖：
1. 作一三角形 ABC；
2. 分別在 AB 和 BC 邊上作垂直平分線；
3. 兩垂直平分線相交於 G 點；
4. 在 CA 邊上作一垂直平分線；
5. 以交點 G 為圓心，OB 為半徑作一圓形。



觀察／實驗：
1. 在 CA 邊上所作的垂直平分線能否通過其餘兩條垂直平分線上的交點 G 呢？
2. 任意拖曳三角形上其中一個頂點，三條垂直平分線會否相交於 G 點呢？
3. 以 GB 為半徑所作的圓會否過頂點 A 和 C 呢？對 GA, GB 和 GC 的長度有甚麼發現？
4. 標註和量度∠ABC, 然後拖曳 B 點，G 點是會跟著移動的，G 點可能在圓周上，或在圓周內或周外，分別觀察這三種情況。

三角形 ABC 三邊的垂直平分線的交點 G 稱為三角形的**外接圓心**，此圓外切於三角形 ABC 稱為**外接圓**。

猜想：
寫出你的假設：(可附加紙張)

<u>討論後再觀察</u>：

1. 與同學討論所得結果，並且向同學介紹你如何建立這些假設。
2. 除三角形外，嘗試將其他圖形外接圓，並且向同學介紹你的嘗試經驗，將你的假設記在下面。

教學指引 (4)

<u>**題目**</u>：發現三角形三邊垂直平分線的性質。

<u>**基本知識**</u>：學生已認識垂直平分線的定義，並認識銳角、直角和鈍角三種三角形。

<u>**教學時間**</u>：約 25 分鐘。

<u>**作圖應注意**</u>：

(a) 在步驟 3 和 4 中，學生應選取兩條合適的垂直平分線上定立交點，才作第三條 垂直平分線；

(b) 在步驟 5 中，學生可能沒有以交點至其中一三角形頂點為半徑而任意作一個通過三角形頂點的圓形，這樣，當學生拖曳三角形其中一個頂點時，圓形可能未必內接三角形。

<u>**探討／假設應注意**</u>：

(a) 學生應察覺到這三條垂直平分線必經過同一點這種特別的性質，同時他們亦要明白三條隨意的線段是未必相交於同一點，所以，學生應認識有關兩條線，三條線或更多線段相交時的交點數目基本知識。

(b) 猜想中可包括：

(1)任意三角形的垂直平分線相交於一點。

(2)外心是與三角形三個頂點等距。

(3)三角形的外心是其外接圓的圓心。

(4)外心是在銳角三角形之內，在直角三角形的斜邊上，及在鈍角三角形之外。

(5)任何半圓上可內接一直角三角形。

(6)直角三角形斜邊是外接圓的直徑。

(c) 當學生作出猜想*(1)*後，便要認識**外心**及**外接圓**名稱。

<u>**討論後和再觀察應注意**</u>：

(a) 學生可嘗試將圓內接其他圖形，如四邊形，他們將發覺並非所有圓形可內接四邊形。

這時教師可指出只有一種特殊的四邊形，叫圓內接四邊形才可被圓形內接，而其相關的性質已在較早前實驗討論過。

(b) 同時教師可引導學生去發現在任意三角形中的中線，角平分線及高都是會相交於一點，可藉這些活動令學生欣賞數學優美之處。

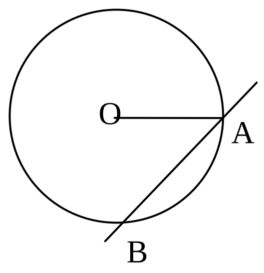
實驗工作紙 (5)

圓的切線

目的：
發現圓的切線性質。

當一直線通過圓形時，此直線與圓可*相交於兩點*或與圓只*接觸於一點*。當直線與圓相交於兩點，該直線稱為**割線**。直線與圓相交於一點，該線就是圓的**切線**，而接觸點稱為**切點**。
利用 Cabri 的簡單指令，在圓上作一條割線，然後將此割線移動而成為該圓的切線。

作圖：



1. 以 O 為圓心，OA 為半徑作一圓形；
2. 作半徑 OA；
3. 在圓上作一 B 點；
4. 過 A,B 兩點作一直線(i.e.割線 AB)
5. 標註和量度∠OAB。

探討：
1. 在圓周上拖曳 B 點逐漸移近 A 點，當 B 點靠近 A 點時，對∠OAB 的大小有甚麼發現？
2. 當 B 點落在 A 點時，割線 AB 即成為該圓的切線，這切線與經過圓心至切點的半徑之間有甚麼關係呢？試運用這關係去找出在圓上作切線的方法。

猜想：
寫出你的假設。

討論後和再觀察
1. 與同學討論所得結果，並且向同學展示你如何建立這些假設。
2. 試找出如何作兩外切或內切圓(兩圓相交於一點)，並將方法及發現記在下面。

教學指引 (5)

<u>題目</u>：發現圓的切線性質。

<u>基本知識</u>：學生已認識*割線*和 *切線*的性質。

<u>教學時間</u>：約 15 分鐘。

<u>作圖應注意</u>：

學生應利用“Line by two points”指令作割線 AB，否則當 B 點落在 A 點時，他們無法看到切線。

<u>探討／假設應注意</u>：

(a) 當拖曳 B 點接近 A 點時，半徑與割線之間所成的角將接近 90º。

(b) 猜想中可包括：

(1) 半徑與切點上的切線成 90º。

(2) 切線與半徑於接觸點互相垂直。

(c) 當學生作出假設時，他們應該明白切線與半徑在圓周上共點。學生並應能在圓上

作出切線，先作半徑，然後通過半徑在圓周上的切點在半徑上作一條垂線。

<u>討論後再觀察應注意</u>：

(a) *作兩外切或內切圓：*

1. 作一圓的切線；
2. 作半徑至切點 X 的延線；
3. 在延線上作一 Y 點；
4. 以 Y 為圓心，XY 為半徑作一圓形；
5. 拖曳 Y 點便可觀察外切圓和內切圓的改變情況。

(b) 教師可展示如何作外切或內切圓形。

Instructional Design 1999

Topic: Circle.

Level: Senior Middle 1 / Form 4.

Subject: Mathematics (Geometry).

Objectives:
1. To study the properties of angles inside a circle and the related theorems.
2. To study the properties of a cyclic quadrilateral and the related theorems.
3. To study the properties of tangents to a circle and the related theorems.
4. To study the circumcircle, etc.

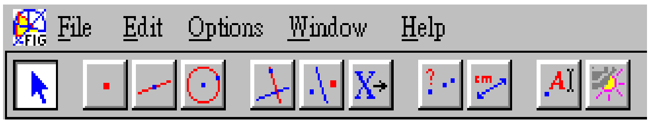
Contents:
By doing Mathematical Experiments and applying the Discovery Learning pedagogy, students should be able to discover :
1. the properties of angles inside a circle.
2. the properties of a cyclic quadrilateral.
3. how to manipulate a secant to become a tangent to a circle, thus they should understand the properties of tangents.
4. the centre of a circle lies on the perpendicular bisectors of the chords.

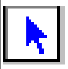
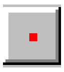
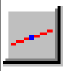
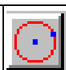
Characteristics:
1. Introduce the basic properties of a circle to students by doing Mathematical Experiments.
2. Attempt to take an empirical study on the pedagogy and the process of learning Mathematics by doing Experiments.
3. Both English and Chinese versions of the Student Activity Sheets and Teaching Notes have been prepared for teachers, this makes it more adaptable for use in English and Chinese sections in Macau schools.

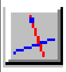
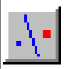
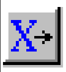
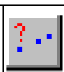
We will use the **Cabri Geometry** software to carry out the activities (Mathematics Experiments). This trial version provided here will terminate in every 15 minutes, but it is enough time for the demonstration in the classroom and for students to work through a series of steps to discover geometric concepts. The **Cabri** package is available in Hong Kong. Product price is HK$1480.

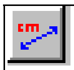
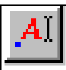
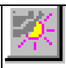
Dealer: Hong Kong Electronic (OA) Centre
G/F 43A Dundas Street
Kowloon
Tel: 23856603

This section is designed to help teachers and students to become accustomed to the **CABRI** software. If you allow your students to carry out the activities on their individual computers, each student should complete the following practice before working on the consecutive activities. This practice allows students to experience the different tools of Cabri and also to reinforce the basic vocabulary they have learned in class. Each activity helps students to discover a theorem, postulate, or geometric concept, and allows students to experience the true meaning of these ideas. Teachers are encourage to make copies of this practice available to all of your students so that they can refer to if they have any problems completing the basic constructions.



 **Pointer** **Rotate** **Dilate** **Rotate and Dilate**	 **Point** **Point On Object** **Intersection Points**	 **Line** **Segment** **Ray** **Vector** **Triangle** **Polygon** **Regular Polygon**	 **Circle** **Arc** **Conic**

 **Perpendicular Line** **Parallel Line** **Midpoint** **Perpendicular Bisector** **Angle Bisector** **Vector Sum** **Compass** **Measurement Transfer** **Locus** **Redefine Object**	 **Reflection** **Symmetry** **Translation** **Rotation** **Dilation** **Inverse**	 **Initial Objects** **Final Objects** **Define Macro**	 **Collinear** **Parallel** **Perpendicular** **Equidistant** **Member**

		
Distance and Length	**Label**	**Hide / Show**
Area	**Comments**	**Colour**
Slope	**Numerical Edit**	**Fill**
Angle	**Mark Angle**	**Thick**
Equation and Coordinates	**Fix / Free**	**Dotted**
Calculate	**Trace On / Off**	**Modify Appearance**
Tabulate	**Animation**	**Show Axes**
	Multiple Animation	**New Axes**
		Define Grid

1. Create and label points.

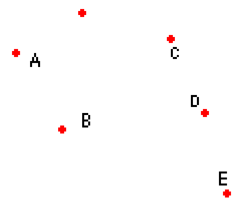


Figure 1.1

1. From the Points Toolbar, select **Point**.
2. Move the pencil to any location in the plane and click. Immediately type the name of the point (for example, A, B, C). Repeat this process to make 5 points.
3. If you forget to label a point, select **Label** from the Display Toolbar. Move the crosshair near a point. When the message ***This point*** appears, click once. A box should appear. Type the label you wish to give this point (for example, A, B, C). Move the crosshair away from the point and click to remove the label box. Repeat this process for each of the points that are still unlabeled.

2. Correct an error.

1. From the Pointer Toolbar, select **Pointer**.
2. Move the crosshair to the point or line you wish to remove. When the appropriate message appears (for example, ***This point***), click to select that figure. (The selected item will begin flashing). Press the DELETE key. The selected figure will disappear.

3. Clear the screen.

1. Click on **Edit** and then click on **Select All**. Press the DELETE key.

   – or –

2. Press and hold the CRTL key and type A. Press the DELETE key.

4. Create two intersecting lines.

1. From the Lines Toolbar, select **Line**.
2. Move the pencil to the plane and click once. Type a point name (for example, A, B, C).
3. Move the pencil to a second spot and click once. This will create a line.
4. Repeat the first three steps to create a second line so that it intersects the first.

5. From the Points Toolbar, select **Intersection Point**.
6. Move the pencil to the intersection point until the message ***Point at this intersection*** appears. Click once. Type the point name.
7. From the Points Toolbar, select **Point On Object**.
8. Move the pencil until the message ***On this line*** appears. Click once. Type the point name.

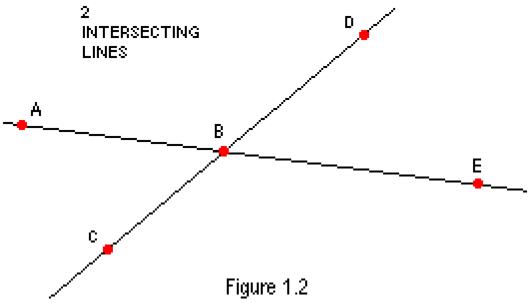


Figure 1.2

5. Clear the screen.

6. Create an angle.

1. From the Lines Toolbar, select **Ray**.
2. Click on the screen where you want the vertex of the angle. Type B.
3. Move the pencil and click once to establish one side of the angle.
4. Move the pencil to the vertex until the message ***This point*** appears. Click once.
5. Move the pencil to create an acute angle and click to establish second side.
6. From the Points Toolbar, select **Point On Object**.
7. Move the pencil to one ray until the message ***On this ray*** appears. Click once. Type A.
8. Move the pencil to the second ray until the message ***On this ray*** appears. Click once. Type C.

7. Measure an angle.

1. From the Measure Toolbar, select **Angle**.
2. Move the cursor to point A. The message ***This point*** appears. Click once.
3. Move the cursor to point B. The message ***This point*** appears. Click once.

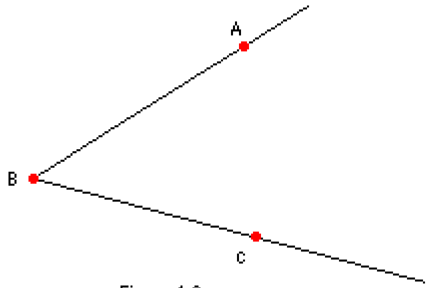


Figure 1.3

4. Move the cursor to point C. The message ***This point*** appears. Click once.
5. The measure of the angle appears with a flashing bar. This allows you to type the name of the angle (for example, ABC = 45.5).

8. Change the angle.

1. From the Pointer Toolbar, select **Pointer**.
2. Move the cross-bar until you see the message **This ray**.
3. Click on one of the rays on the angle and hold until it becomes a hand.
4. You can now drag the angle and watch the angle change.

9. Bisect an angle.

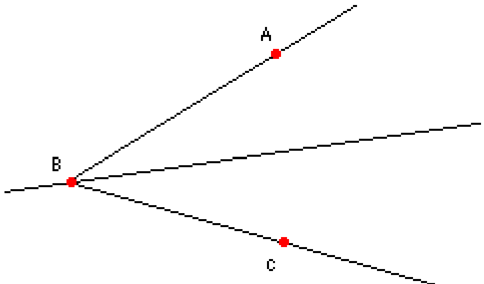


Figure 1.4

1. From the Construct Toolbar, select **Angle Bisector**.
2. Move the cursor to point A. The message ***This point*** appears. Click once.
3. Move the cursor to point B. The message ***This point*** appears. Click once.
4. Move the cursor to point C. The message ***This point*** appears. Click once.
5. Place a point on the bisector in the interior of the angle.
6. Using the new point, measure one of the angles created by the bisector.
7. This measure should be half of the original.

10. Clear the screen.

11. Create parallel lines.

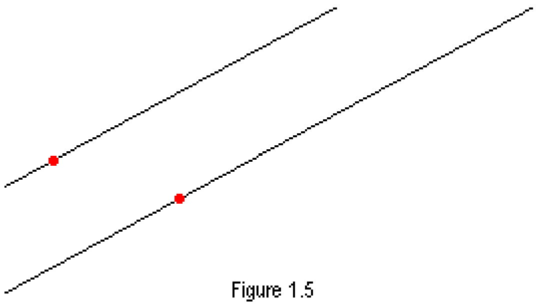


Figure 1.5

1. From the Lines Toolbar, select **Line**.
2. Move the pencil to the plane and click once.
3. Move the pencil to a second spot and click once. This will create a line.
4. From the Construct Toolbar, select **Parallel Lines**.
5. Move the pencil to the line until the message ***Parallel to this line***, appears. Click once.
6. Move the pencil off the line and click once. This creates the parallel line.

12. Clear the screen.

13. Create perpendicular lines.

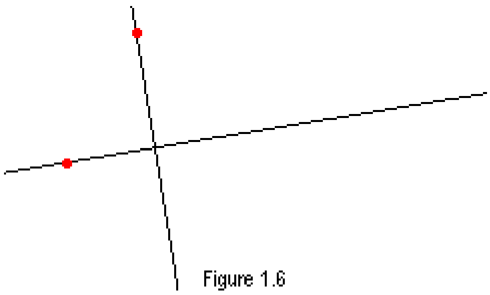


Figure 1.6

1. From the Lines Toolbar, select **Line**.
2. Move the pencil to the plane and click once.
3. Move the pencil to a second spot and click once. This creates a line.

4. From the Construct Toolbar, select **Perpendicular Lines**.
5. Move the pencil to the line until the message ***Perpendicular to this line*** appears. Click once.
6. Move the pencil off the line and click once. This creates the perpendicular line.

14. Clear the screen.

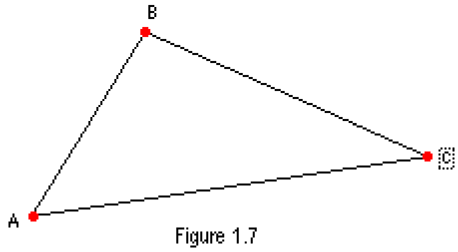


Figure 1.7

15. Create and label a triangle.

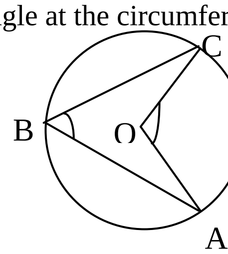
1. From the Lines Toolbar, select **Triangle**.
2. Click once, type A, and drag on the screen.
3. Click again to end the segment and create one side of the triangle. Type B, drag, and click again to finish the triangle. Type C.

STUDENT ACTIVITY SHEET (1)

The Angle at the Centre and The Angle at the Circumference

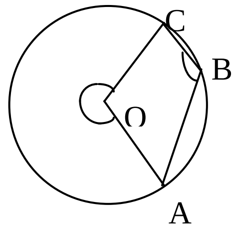
Objective

To discover the relationship between the angle at the centre and the angle at the circumference of a circle.



Construction

1. Construct a basic circle.
2. Construct any three points, A , B and C, on the circle.
3. Construct the centre, O, of the circle.
4. Join the line segments OA , OC , AB and BC .
5. Mark and measure ∠AOC and ∠ABC .



∠AOC is called the **angle at the centre** and ∠ABC is the corresponding **angle at the circumference**.

Investigation

1. Drag the points A, B and C along the circumference of the circle. Observe the changes and relationship between ∠AOC and ∠ABC . Tabulate your results.

∠AOC	20°										
∠ABC	10°										

2. What can you say about the relationship between ∠AOC and ∠ABC ? What can you say when AOC becomes a straight line ?

Conjectures

Write down your conjectures below.

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<u>Discussion</u>

Discuss your results with your group.

<u>Proof</u>

Given : ∠AOC is called the angle at the centre and ∠ABC is the corresponding angle at the circumference.

To prove : ∠AOC = 2 ∠ABC .

TEACHING NOTES (1)

<u>Topic</u> : **The Angle at the Centre and The Angle at the Circumference : Investigation**

<u>Prerequisites</u> : Students should know the meaning of *angle at the centre* and *angle at the circumference*.

<u>Expected time</u> : 15 minutes

<u>Construction notes</u> :

Cabri cannot measure the reflex ∠AOC, so it can be found by 360° – ∠AOC.

<u>Investigation/Conjecture/Discussion notes</u> :

(a) As Cabri can only measure quantities correcting to a certain degree, students may have their conjectures that the angle at the centre is approximately twice the angle at the circumference. In this case, teacher should explain to students that approximation takes place in many situations of our daily life. Teacher may explain why the relation seems to be approximate by an example of ∠AOC being 21.2° and ∠ABC being 10.6°.

(b) Students should be asked to check their result for the case when ∠AOC is a reflex angle.

(c) Some conjectures might be:

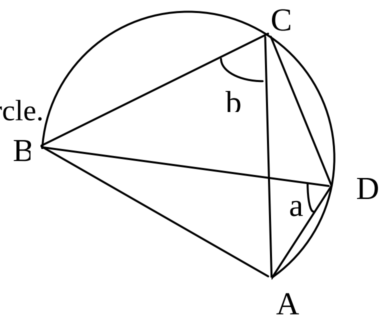
(1) The angle at the centre is twice the angle at the circumference.

(2) If the angle at the centre is 180°, the angle at the circumference is a right angle.

(3) The angle in a semi-circle is a right angle.

STUDENT ACTIVITY SHEET (2)

The Angles in the Same Segment



Objective

To discover the relationship between the angles in the same segment of a circle.

∠ADB and ∠ACB are called the **angles in the same segment.**

Construction

1. Construct a basic circle.
2. Construct any four points, A , B , C and D, on the circle.
3. Join the line segments AB, BC , CD , DA , BD and AC .
4. Mark and measure the eight angles in the diagram.

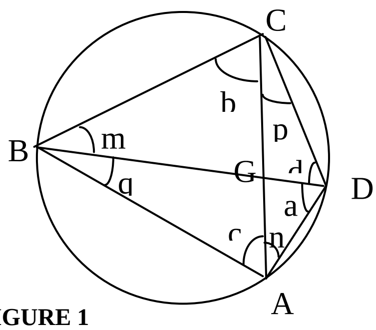


FIGURE 1

Investigation

1. Drag the points A, B , C and D along the circumference of the circle. Observe the changes and relationship between the eight marked angles. Tabulate your results.

a	60°										
b	60°										
c	55°										
d	55°										
m	35°										
n	35°										
p	30°										
q	30°										

2. What can you say about the relationship among these eight angles? What conclusion can you draw ?

Conjectures

Write down your conjectures below.

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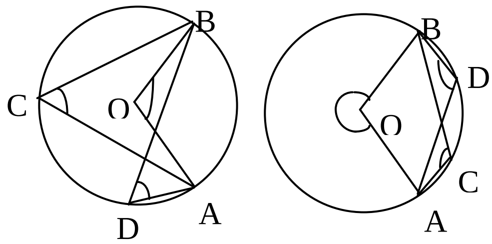
<u>Discussion and further investigation</u>

1. Discuss your results with your group. Can you prove your conjecture by the property that angle at the centre is twice the angle at the circumference? Write down your proof below.



2. Investigate if there is any pair of similar triangles in FIGURE 1.

TEACHING NOTES (2)

<u>Topic</u> : **The Angles in the Same Segment : Investigation**

<u>Prerequisites</u> : Students should know the meaning of *angles in the same segment*.

<u>Expected time</u> : 20 minutes

<u>Investigation/Conjecture notes</u> :

(a) Some conjectures might be:

(1) The angles in the same segment are equal.

(2) $a = b$, $c = d$, $p = q$, $m = n$.

(b) Some students might also have the following conjectures:

(1) $a + b + c + d + p + q + m + n = 360°$.

(2) $a + d + m + q = 180°$, $b + c + n + p = 180°$.

Students may be asked to investigate if *(1)* and *(2)* are true for a general quadrilateral. They should be able to see that *(2)* is not true for a general quadrilateral.

<u>Discussion and further investigation notes</u> :

(a) The proof is as follows:

$\angle AOB = 2\angle ACB$ ($\angle$. at centre twice $\angle$. at circumference)

$\angle AOB = 2\angle ADB$ ($\angle$. at centre twice $\angle$. at circumference)

Hence, $\angle ACB = \angle ADB$.

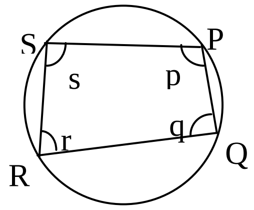
(b) Teacher may ask students to investigate if there is any pair of similar triangles in the first figures. There are two pairs of similar triangles, i.e. $\Delta BGC \approx \Delta AGD$ and $\Delta BGA \approx \Delta CGD$.

STUDENT ACTIVITY SHEET (3)

Cyclic Quadrilateral

Objective

To discover the properties of a cyclic quadrilateral.



Construction

1. Construct a basic circle.
2. Construct any four points, P , Q , R and S, on the circle.
3. Join the line segments PQ , QR , RS and SP .
4. Mark and measure p , q , r and s .

A **cyclic quadrilateral** is a quadrilateral whose vertices lying on the same circle.

Investigation

1. Drag the points P, Q , R and S along the circumference of the circle. Observe the changes and relationship between angles p , q , r and s . Tabulate your results.

p	107°							
q	102°							
r	73°							
s	78°							

2. What can you say about the relationship between p , q , r and s ? Are these results true for a general quadrilateral ? What can you say about a cyclic quadrilateral ?

Conjectures

Write down your conjectures below.

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Discussion and further investigation

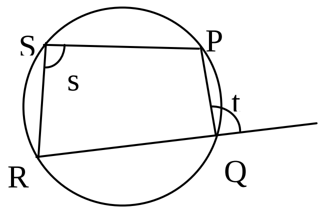
1. Discuss your results with your group.
2. What can you say about the relationship of the two marked angles in the figure below? Can you explain your conjecture using the above results. Check your conjecture using

Cabri.

3. Given : PQRS is a cyclic quadrilateral.

   To prove: s = t.

   Proof :



TEACHING NOTES (3)

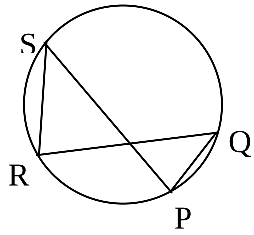
Topic : **Cyclic Quadrilateral : Investigation**

Prerequisites : Students should know the meaning of *a cyclic quadrilateral.*

Expected time : 20 minutes

Investigation/Conjecture notes :

(a) If the points P, Q , R and S are dragged to the case that PQRS is not a cyclic quadrilateral (see figure on the right), then it will be a case showing angles in the same segment are the same.



(b) Some conjectures might be:

(1) $p + q + r + s = 360°$.

(2) $p + r = 180°$, $q + s = 180°$.

(3) Opposite angles of a cyclic quadrilateral are supplementary.

Students may be asked to investigate if *(1)* , *(2)* and *(3)* are true for a general quadrilateral. They should be able to see that *(2)* and *(3)* are not true for a general quadrilateral. Teacher may then emhpasize that cyclic quadrilateral is one kind of special quadrilaterals which have many properties.

Discussion and further investigation note :

Student should be able to explain why the exterior angle (t) is equal to the interior opposite angle (s) in a cyclic quadrilateral by using the properties :

(i) opposite angles of a cyclic quadrilateral are supplementary ;

(ii) adjacent angles on a straight line are supplementary.

STUDENT ACTIVITY SHEET (4)

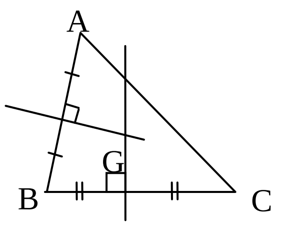
The Circumcircle of a Triangle : Investigation

Objective

To discover the properties of perpendicular bisectors in a triangle.

Using the basic commands of Cabri, you are required to construct a circle passing through the vertices of a triangle.

Construction



1. Construct a triangle ABC .
2. Construct the perpendicular bisectors of sides AB and BC.
3. Construct the intersection, G, of the two perpendicular bisectors .
4. Construct the perpendicular bisector of side CA .
5. Construct a circle with centre G and radius point B .

Investigation

1. When you constructed the perpendicular bisector of side CA , did it pass through the point of intersection, G, of the other two perpendicular bisectors?
2. Drag any vertex of the triangle. Do these perpendicular bisectors *always* meet at the same point G ?
3. Does the circle GB always pass through the vertices A and C ? What can you say about the lengths GA , GB and GC ?
4. Mark and measure $\angle ABC$, and then drag the point B. The point G will move accordingly and it may fall inside, on or outside the triangle. Investigate these three cases.

The point of concurrency, G , of the three perpendicular bisectors is called the **circumcentre** of the triangle ABC . The circle GA is called the **circumcircle** of the triangle ABC .

Conjectures

Write down your conjectures below. (Use additional sheets if necessary.)

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Discussion and further investigation

1. Discuss your results with your group. Present your findings to your classmates showing how you arrived at your conjectures.
2. Try to circumscribe other shapes besides triangles. Describe how you try and write down any conjectures that you can come up with.

TEACHING NOTES (4)

<u>Topic</u> : **The Circumcircle of a Triangle : Investigation**

<u>Prerequisites</u> : Students should know the meaning of *perpendicular bisector*. They should know how to classify a triangle as acute-angled, right-angled or obtuse-angled.

<u>Expected time</u> : 25 minutes

<u>Construction notes</u> :

(a) In Step 3-4, if the three perpendicular bisectors are constructed before the point of intersection is constructed, students will need to select the two appropriate lines to form the point of intersection.

(b) In Step 5, students may simply construct an arbitrary circle that goes through the vertices of the triangle without actually using one of the vertices as the radius point. In this case, the circle may not circumscribe the triangle when they drag the points of the triangle.

<u>Investigation/Conjecture notes</u> :

(a) Students should observe that the three perpendicular bisector always meet at the same point. They may not find this fact unusual unless they have thought about the chances of three random lines meeting at the same point. An exercise of investigating points of intersection of two, three and more lines might be a good pre-activity.

(b) Some conjectures might be:

(1) The perpendicular bisectors in any triangle meet at the same point (concurrent).

(2) The circumcentre is equidistant from the three vertices of a triangle.

(3) The circumcentre of a triangle is the centre of the circumcircle.

(4) The circumcentre lies inside an acute-angled triangle, on the hypotenuse of a right-angled triangle, and outside an obtuse-angled triangle.

(5) Any triangle inscribed in a semi-circle is a right-angled triangle.

(6) The hypotenuse of a right-angled triangle is a diameter of the circumcircle.

(c) Introduce the terms *circumcentre* and *cirumcircle* after students have made conjecture *(1)*.

<u>Discussion and further investigation notes</u> :

(a) Students can try to circumscribe other shapes, like quadrilateral. They will be able to discover that not all quadrilaterals can be circumscribed by a circle. Teachers may reinforce students that those can be circumscribed is a special kind of quadrilaterals, called cyclic quadrilaterals, which have many properties discussed before.

(b) Teacher may ask students to investigate about the concurrencies of medians, angle bisectors or altitudes of any triangle, so that students may have the chance to appreciate the beauty of Mathematics.

STUDENT ACTIVITY SHEET (5)

Tangents to a Circle : Investigation

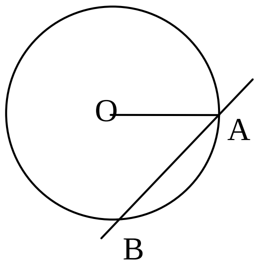
Objective

To discover the properties of tangents to a circle.

When a straight line intersects a circle, it either *cuts* the circle at *two* distinct points or *touches* the circle at only *one* point. A line that intersects a circle at two points is called a **secant**. A line that intersects a circle at only one point is called a **tangent** to the circle and the point of intersection is called the **point of contact**.

Using the basic commands of Cabri, you are required to construct a secant and then to manipulate it until it becomes a tangent.

Construction



1. Construct a circle with centre O and radius point A .
2. Construct the radius OA .
3. Construct a point B on the circle.
4. Construct a line passing through A and B (i.e. secant AB).
5. Mark and measure $\angle OAB$.

Investigation

1. Drag the point B around the circle towards the point A . What happens to $\angle OAB$ as the point B gets closer and closer to A ?
2. When the point B coincides with the point A , the secant AB becomes a tangent to the circle. What is relationship between a tangent and a radius from centre to the point of contact? Use this relationship to devise a method of constructing a tangent to the circle.

Conjectures

Write down your conjectures below.

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Discussion and further investigation

1. Discuss your results with your group. Present your findings to your classmates showing how you arrived at your conjectures.
2. Try to devise method(s) for constructing externally or internally tangent circles (circles that intersect at only one point). Describe your method(s) and write down any discoveries that you can come up with these circles.

TEACHING NOTES (5)

Topic : **Tangents to A Circle : Investigation**

Prerequisites : Students should know the meaning of *secant* and *tangent*.

Expected time : 15 minutes

Construction note :

It is better if students draw the secant AB using the command "Line by two points", rather than "Segment", otherwise they will not be able to see the tangent when B and A coincide.

Investigation/Conjecture notes :

(a) When the point B is dragged towards A, the angle between the radius and the secant approaches 90°.

(b) Some conjectures might be:

(1) The angle between a radius and the tangent through the radius point is 90°.

(2) A tangent to a circle is perpendicular to the radius at the point of contact.

(c) Make sure when students make their conjectures, they know that the tangent and the radius share a common point on the circumference of the circle. They should be able to construct a tangent to the circle, by first drawing a radius and then a line perpendicular to the radius and passing through the intersection point between the radius and the circle.

Discussion/Further Investigation notes :

(a) *Construction of externally or internally tangent circles* :

1. draw a tangent to a circle ;
2. extend the radius at the point of contact, X , to a full line ;
3. construct a point, Y , on this extended line ;
4. construct a circle using Y as the centre and X as the radius point ;
5. drag Y to see when externally or internally tangent circle will be formed.

(b) Teacher may demonstrate how to draw externally or internally tangent circles.